# Homework 5

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**Problem 10:** Minimize the total completion time of a set of jobs.

(a) **Theorem:** The $SJF$ algorithm is incorrect.

**Proof:** Let $J$ be a collection of two jobs $\{(0,10),(5,8)\}$ where each job $J_i$ is a tuple $(r_i, x_i)$ of non-negative integers specifying the release time and size of the job respectively.

According to $SJF$, the schedule is as follows: between time 0 and time 5, $J_1$ is the only job that is released; thus, it will run. At time 5, $J_1$ will be preempted for $J_2$ because $x_2 < x_1$. That is, the size of $J_2$, 8, is less than the size of $J_1$, 10. $J_2$ will run from time 5 to time 13. At time 13, $J_2$ will have completed, and $J_1$ will finish running between time 13 and time 18.

Thus, the total completion time $C = C_1 + C_2 = 18 + 13 = 31$.

However, the following schedule has a smaller completion time: $J_1$ runs between time 0 and time 10 (it does not get preempted at time 5). Then, $J_2$ runs between time 10 and time 18. The total completion time $C = C_1 + C_2 = 10 + 18 = 28$.

Therefore, $SJF$ is clearly incorrect.

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(b) **Theorem:** The greedy algorithm $S$ correctly solves this problem.

**Proof:** Assume to reach a contradiction that $S$ is not correct. Thus, there exists an input $I$ on which $S$ does not produce an optimal solution. Let the acceptable output produced by $S$ be $S(I)$. That is, $S(I)$ is a sequence of $n$ time intervals $S(I) = \{j_{S1}, j_{S2}, \ldots, j_{Sn}\}$ where $j_{Si}$ represents the job that is running at time $i$.

Let $O(I) = \{j_{O1}, j_{O2}, \ldots, j_{On}\}$ where $j_{Oi}$ represents the job that is running during time $i$. Let this be the optimal solution that minimizes the completion time agrees with $S(I)$ for the most number of steps.

Let $j_k$ be the first time interval in which $S$ and $O$ select different jobs to be run.

Note that $O(I)$ must run and complete $j_{Sk}$ at some point after time $k$ because it is feasible.

Consider the set $O'(I)$ that is constructed from $O(I)$ by swapping all occurences of $j_{Ok}$ with $j_{Sk}$ from time $k$ and on.

That is, the first occurence of $j_{Sk}$ in $O(I)$ after time $k$ is swapped with $j_{Ok}$ at time $k$. The next occurence of $j_{Sk}$ in $O(I)$ is swapped with the next occurence of $j_{Ok}$. This process continues for the number of consecutive intervals in which $S(I)$ runs $j_{Sk}$.

Note that the number of consecutive intervals in which $S(I)$ runs $j_{Sk}$ must be less than or equal to the number of occurences of $j_{Ok}$ in $O(I)$ by definition of the greedy algorithm.

Thus, $O'(I)$ agrees with $S(I)$ in at least one more step, forming a contradiction. Therefore, the algorithm $S$ is correct.

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